Dynamics 4

Learning Module 2

Lagrange's method for non-conservative systems

THE UNIVERSITY *of* EDINBURGH
School of Engineering

DYNAMICS 4

LM2 GUIDE 26 27

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Learning Module 2 | Generalised forces and non-conservative systems

Estimated time: 10 hours Complete module before: 13 October

1. Start by reading the Course Notes: For this module you should read **Section 2.3.** Take hand-written notes as you read.

2. Attend the lectures and tutorial sessions and, if you need, watch the lecture recordings later. This will help consolidate what you learned. Keep taking notes.

3. Attempt to solve **Questions 2.1 to 2.5** in **Exercise Sheet 2**. Compare your solutions to the numerical solutions provided at the end of each question. Write down any questions you might have.

4. Compare your solutions to the worked-out example solutions to **Questions 2.1 to 2.5**. Discuss any questions you might have in the tutorial sessions. Arrange to meet with your lecturer if you need further clarifications.

5. Be active in the Learn discussion board thread for this Learning Module, which is labeled **analytical_dynamics**. You can post your own questions and help clarify or answer the questions of others.

6. You may want to read **Section 2.6** of the Course Notes with examples of real life applications.



In this module

Generalised forces
Generalised form of Lagrange's equations
Virtual displacements and virtual work
Damping forces
Other generalised forces

Learning outcomes

Apply virtual work-based methods to dynamical systems, relating between Lagrangian and Newtonian mechanics

Derive equations of motion for dynamic systems from energy-based approaches (e.g. Lagrange's method)

This is a study and learning guide to Learning Module 2. You can find the corresponding lecture recordings and slides on Learn. This module has a dedicated thread in the discussion board on Learn.

Please note that the course time and the recommended *before dates*, are estimated and may differ from person to person. You should not see these as targets but instead as a way to help you plan your semester and the preparation for exams. The estimated time includes going through all the course materials, interaction in the discussion boards and self-study time.

Lagrange's equations

- Conservative systems (energy does not change)
- i.e. no friction/damping and no external forces doing work
- For a n-degree-of-freedom (n-d.o.f.) system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0 \quad (j = 1, 2, \ldots, n)$$

where:

$q_1, q_2, \ldots, q_n$	are the generalised coordinates
$T(q_j, \dot{q}_j)$	is the kinetic energy
$V(q_j)$	is the potential energy

Lagrange's equations

- Non-conservative systems (e.g. with friction/damping)
- Generalised forces
- For a n-degree-of-freedom (n-d.o.f.) system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = Q_j \quad (j = 1, 2, \ldots, n)$$

where:

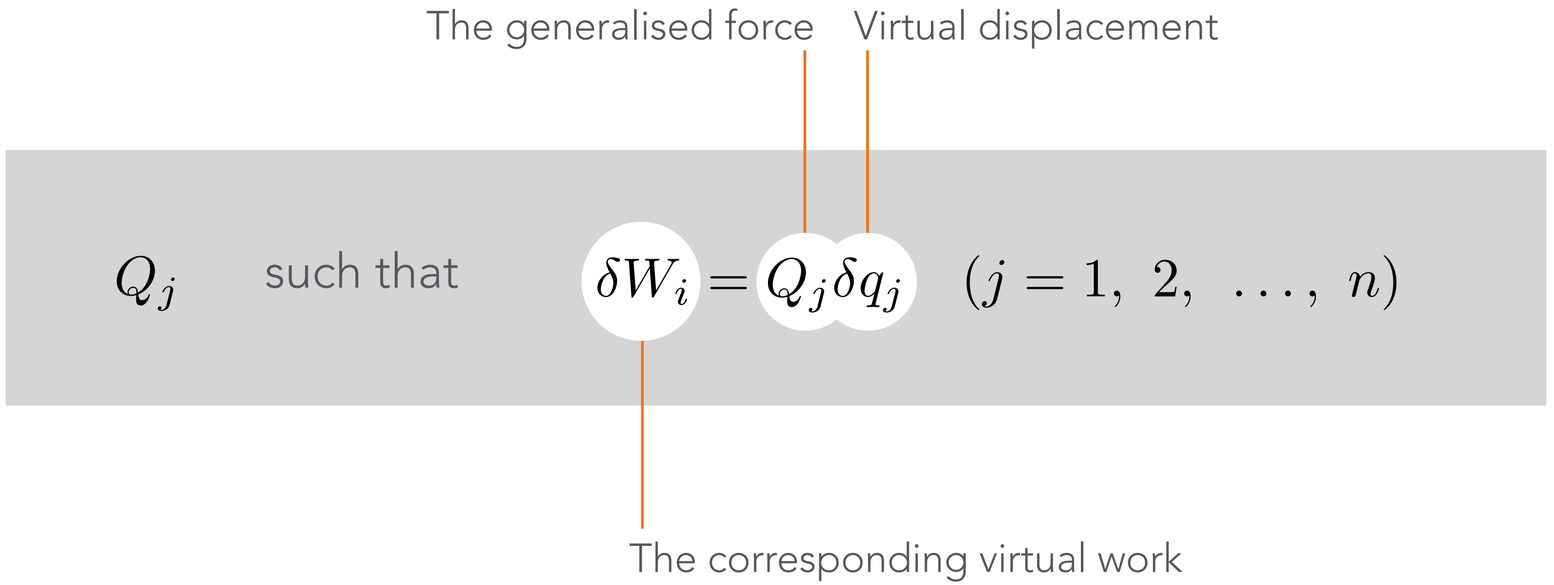
$q_1, q_2, \ldots, q_n$ are the generalised coordinates

$T(q_j, \dot{q}_j),\ V(q_j)$ are the kinetic and potential energies

$Q_1, Q_2, \ldots, Q_n$ are the generalised forces

What is a generalised force?

It is determined from the virtual work done by the non-conservative forces during the virtual displacement of each coordinate.



What is a generalised force?

It is determined from the virtual work done by the non-conservative forces during the virtual displacement of each coordinate.

Generalised force

$$Q_j = \frac{\delta W_i}{\delta q_j} \quad (j = 1,\ 2,\ \ldots,\ n)$$

What is a virtual displacement?

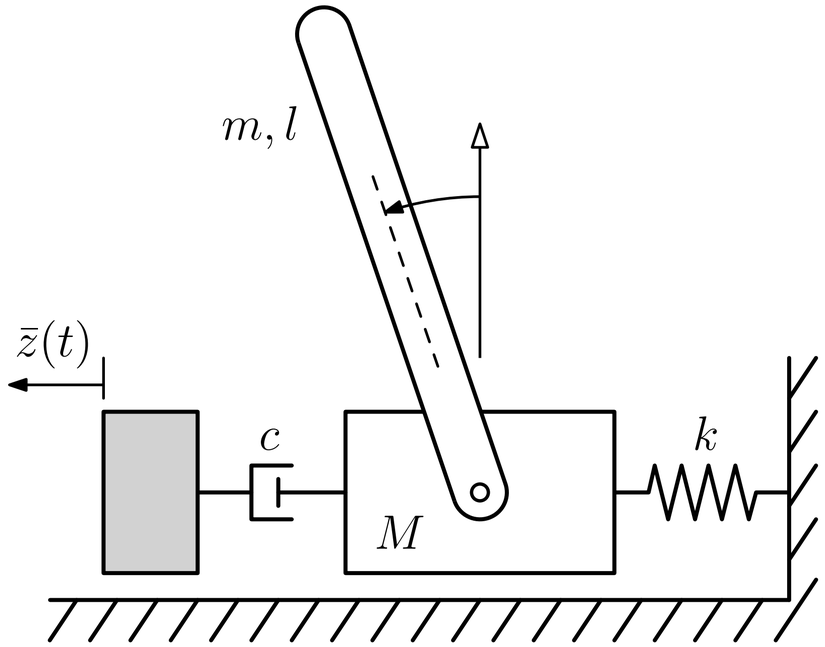
- A mental exercise.
- Must be possible (i.e. not violate constraints).
- Associated to each generalised coordinate (d.o.f.)

Example (CN)

Consider the system shown:

a) Identify a set of coordinates (d.o.f.) for the system shown.

b) Find the equations of motion using Lagrange's method.



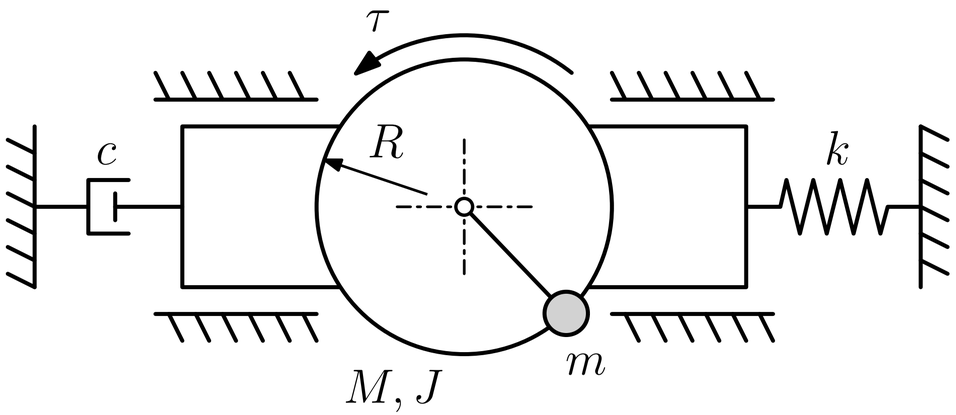
Example (CN)

Consider the unbalanced vertical axis rotor shown:

a) Identify a set of generalised coordinates (d.o.f.) for the system.

b) Identify the corresponding generalised forces.

c) Obtain the equations of motion using Lagrange's method.



Example (CN)

Consider the system shown:

a) Identify the corresponding generalised forces.

b) Find the equations of motion using Lagrange's method.

